DARK MATTER HALOS IN A SECONDARY INFALL MODEL

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ABSTRACT

We calculate the density profiles of virialized halos in the case of structure evolving hierarchically from a scale-free Gaussian $\delta-$field having a power spectrum $P(k) \propto k^n$ in a $\Omega = 1$ Universe; we suppose that the initial density contrast profile around local maxima is given by the mean peak profile introduced by Bardeen et al. (1986 hereafter BBKS). We show both that the density profiles are not power-laws but have a logarithmic slope that increases from the inner halo to its outer parts and for $n \geq -1$ are well approximated by Navarro et al. (1995, 1996, 1997) profile and the radius a, at which the slope $\alpha = -2$, is a function of the mass of the halo and of the spectral index n.

1 Introduction

The collapse of perturbations onto local density maxima of the primordial density field is likely to have played a key role in the formation of galaxies and clusters of galaxies. The problem of the collapse has been investigated from several authors (Gunn & Gott 1972; Gunn 1977; Kaiser 1984; Davis et al. 1985; Hoffman & Shaham 1985 - hereafter HS; BBKS; Hoffman 1988; Efstathiou et al. 1988; Evrard et al. 1993; Navarro et al. 1995, 1996, 1997; Avila-Reese et al. 1998).
To overcome the problem of the excessively steep density profiles, $\rho \propto r^{-4}$, obtained

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in numerical experiments of simple gravitational collapse Gunn & Gott (1972), Gott (1975) and Gunn (1977) were able to produce shallower profiles, $\rho \propto r^{-2}$ through the *secondary infall* process. HS considered a scale-free initial perturbation spectra, $P(k) \propto k^n$ and assumed that local density extrema are the progenitors of cosmic structures and that the density contrast profile around maxima is proportional to the two-point correlation function. They thus showed that $\rho \propto r^{-\alpha}$ with $\alpha = \frac{3(3+n)}{(4+n)}$, thus recovering Bertschinger's (1985) profile for $n = 0$ and $\Omega = 1$. They also showed that, in an open Universe, the slopes of the density profiles steepen with increasing values of n and with decreasing Ω, reaching a profile $\rho \propto r^{-4}$ for $\Omega \rightarrow 0$. Hoffman (1988) refined the calculations by HS and made a detailed comparison of the analytical predictions of the *Secondary Infall Model* (hereafter SIM) with the simulations by Quinn et al. (1986) and Quinn & Zurek (1988). The good results given by the SIM in describing the formation of dark matter halos seem to be due to the fact that in energy space the collapse is ordered and gentle, differently from what seen in N-body simulations (Zaroubi et al. 1996).
A great effort has been dedicated to study the role of initial conditions in shaping the final structure of the dark matter halos; but, if on large scales (evolution in the weakly non-linear regime) the growing mode of the initial density fluctuations can be recovered if the present velocity or density field is given (Peebles 1989; Nusser & Dekel 1992), on small scales shell crossing and virialization contribute to make the situation less clear. To study the problem, three-dimensional large-scale structure simulations were run with often conflicting results (Quinn et al. 1986; West et al. 1987; Efstathiou et al. 1988). More recent studies (Voglis et al. 1995; Zaroubi et al. 1996) showed a correlation between the profiles and the final structures. Besides Lemson (1995), Cole & Lacey (1996), Navarro et al. (1996, 1997) and Moore et al. (1997) found that dark matter halos do not follow a power law but develop universal profile, a quite general profile for any scenario in which structures form due to hierarchical clustering, characterized by a slope $\beta = \frac{d \ln \rho}{d ln r} = -1$ near the halo center and $\beta = -3$ at large radii. In that approach, density profiles can be fitted with a one parameter functional form:

$$\frac{\rho(r)}{\rho_b} = \frac{\delta_n}{\frac{r}{a}\left(1 + \frac{r}{a}\right)^2} \tag{1}$$

where ρ_b is the background density and δ_n is the overdensity [below we shall refer to Eq. (1) (Navarro et al. 1997) as the NFW profile]. The scale radius a, which defines the scale where the profile shape changes from slope $\beta < -2$ to $\beta > -2$, and the characteristic overdensity, δ_n, are related because the mean overdensity enclosed

within the virial radius r_{vir} is $\simeq 180$. The scale radius and the central overdensity are directly related to the formation time of a given halo (Navarro et al. 1997). The power spectrum and the cosmological parameters only enter to determine the typical formation epoch of a halo of a given mass, and thereby the dependence of the characteristic radius on the total mass of the halo. Also these last results are not universally accepted. In short, the question of whether galaxies and clusters mass density profiles retain information on the initial conditions and the evolutionary history that led to their formation remains an open question.
In this paper, we introduce a modified version by HS and Hoffman's (1988) models to study the shapes of the density profiles that result from the gravitational collapse. In particular, we relax the hypothesis that the initial density profile is proportional to the two-point correlation function, and use the density profiles given by BBKS. The plan of the paper is the following: in Sect. 2 we introduce our model and in Sect. 3 we show our results and we draw our conclusions.

2 The model

In the most promising cosmological scenarios, structure formation in the universe is generated through the growth and collapse of primeval density perturbations originated from quantum fluctuations (Guth & Pi 1982; Hawking 1982; Starobinsky 1982; BBKS) in an inflationary phase of early Universe. The growth in time of small perturbations is due to gravitational instability. The statistics of density fluctuations originated in the inflationary era are Gaussian, and can be expressed entirely in terms of the power spectrum of the density fluctuation sP(k). In biased structure formation theory it is assumed that cosmic structures of linear scale R_f form around the peaks of the density field, $\delta(r)$, smoothed on the same scale.
If we suppose we are sitting on a $\nu\sigma$ extremum in the the smoothed density field, we have that:

$$\delta(0) = \nu\xi(0)^{1/2} = \nu\sigma \tag{2}$$

together with:

$$\bigtriangledown\delta(r) \mid_{r=0} = 0 \tag{3}$$

where $\xi(r)$ is the two-point correlation function given in $r = 0$ If the Laplacian of $\delta(r)$ is unspecified, that means that the extremum may be a maximum or a minimum, the mean density at a distance r from the peak is then:

$$\delta(r) = \nu\xi(r)/\xi(0)^{1/2} \tag{4}$$

(Peebles 1984; HS). If we calculate the mean density around maxima, as done by BBKS, by adding the constraint:

$$\bigtriangledown^2 \delta(r) \mid_{r=0} < 0 \tag{5}$$

we find that the mean density around a peak is given by:

$$\langle \delta(r) \rangle = \frac{\nu \xi(r)}{\xi(0)^{1/2}} - \frac{\vartheta(\nu\gamma, \gamma)}{\gamma(1-\gamma^2)} \left[\gamma^2 \xi(r) + \frac{R_*^2}{3} \nabla^2 \xi(r) \right] \cdot \xi(0)^{-1/2} \tag{6}$$

(BBKS; Ryden & Gunn 1987), where ν is the height of a density peak, γ and R_* are two spectral parameters while $\vartheta(\gamma\nu, \gamma)$ is:

$$\theta(\nu\gamma, \gamma) = \frac{3(1-\gamma^2) + (1.216 - 0.9\gamma^4) \exp\left[-\left(\frac{\gamma}{2}\right) \left(\frac{\nu\gamma}{2}\right)^2 \right]}{\left[3\left(1-\gamma^2\right) + 0.45 + \left(\frac{\nu\gamma}{2}\right)^2 \right]^{1/2} + \frac{\nu\gamma}{2}} \tag{7}$$

In order to calculate $\delta(r)$ we need a power spectrum, $P(k)$. In the following, we restrict our study to an Einstein-De Sitter ($\Omega = 1$) Universe with zero cosmological constant and scale-free density perturbation spectrum $P(k)$

$$P(k) = Ak^n \tag{8}$$

with a spectral index in the range $-1 \leq n \leq 0$, and also to a CDM Universe with spectrum given by BBKS. We normalized the spectrum by imposing that the mass variance at $8h^{-1}Mpc$ is $\sigma_8 = 0.63$. In the case of a free-scale power spectrum, it is easy to show that the two-point correlation function can be expressed in terms of the confluent hypergeometric function, ${}_1F_1$, and of the Γ (see Del Popolo et al. 1999) function. In the case that ν is very large Eq. (7) reduces to

$$\theta \rightarrow 3(1-\gamma^2)/(\nu\gamma) \tag{9}$$

and the mean density is well approximated by Eq. (4), which is the approximation used by HS to calculate $\delta(r)$. In reality, for peaks having $\nu = 2, 3, 4$, the mean expected density profile is different from a profile proportional to the correlation function both for galaxies and clusters of galaxies (see BBKS). For example for galaxies the CDM profile is steeper than that proportional to $\xi(r)$ as shown by Ryden & Gunn (1987) with a discrepancy increasing with decreasing ν. As shown

by Gunn & Gott (1972), a bound mass shell having initial comoving radius x will expand to a maximum radius:

$$r_m = x/\overline{\delta}(r) \tag{10}$$

where the mean fractional density excess inside the shell, as measured at current epoch t_0, assuming linear growth, can be calculated as:

$$\overline{\delta} = \frac{3}{r^3}\int_0^r \delta(y) y^2 dy \tag{11}$$

At initial time t_i and for a Universe with density parameter Ω_i, a more general form of Eq. (10) (Peebles 1980) is :

$$r_m = r_i \frac{1+\overline{\delta}_i}{\overline{\delta}_i - (\Omega_i^{-1} - 1)} \tag{12}$$

The last equation must be regarded as the main essence of the SIM. It tells us that the final time averaged radius of a given Lagrangian shell does scale with its initial radius. Expressing the scaling of the final radius, r, with the initial one by relating r to the turn around radius, r_m, it is possible to write:

$$r = F r_m \tag{13}$$

where F is a costant that depends on α. (Zaroubi et al. 1996). If energy is conserved, then the shape of the density profile at maximum of expansion is conserved after the virialization, and is given by (Peebles 1980; HS; White & Zaritsky 1992):

$$\rho(r) = \rho_i \left(\frac{r_i}{r}\right)^2 \frac{dr_i}{dr} \tag{14}$$

In the limit $\nu >> 1$, the overdensity $\delta(r)$ is proportional to the two-point correlation function and the density profile is a function of n and Ω only, and then the expected profile is that by HS.

3 Results and conclusions

By using the model introduced in the previous section we have studied the density profiles of halos in free-scale universes with $-1 \leq n \leq 0$, and for a CDM model characterized by a BBKS spectrum. As previously quoted, the chosen range of n is dictated by the limits of the SIM and by the values of n interesting in the

cosmological context.
In Fig. 1 we plot the slope of the density profile for several values of n and ν. We began by finding the HS solution, that was recovered as expected in the limit $\nu >> 1$. This solution is represented by the short-dashed line which coincides with the result by HS, namely $\alpha = \frac{3(3+n)}{(4+n)}$, indicating an increase in the slope α with increasing n. Because of the rarity of extremely high peaks, most galaxies and clusters will form

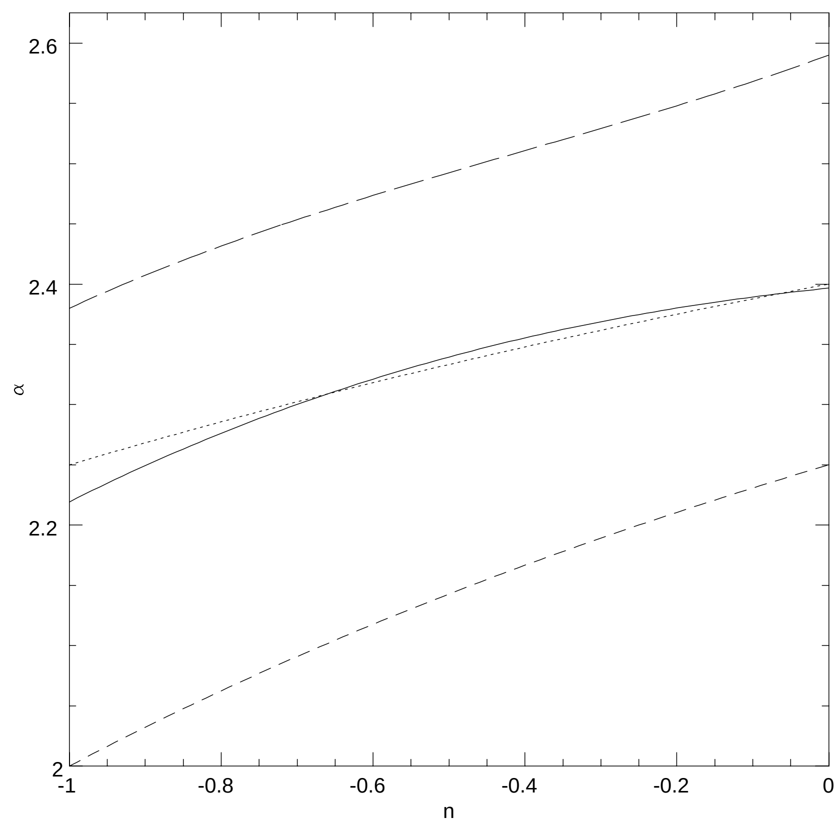

Figure 1: The slope α of density profiles as a function of the spectral index n and ν. The short-dashed line represents α in the limit $n >> 1$. It coincides with the HS result. The solid line represents the logarithmic slope for $\nu = 3$, while the dotted line is Shet & Jain's (1996) result. The long-dashed line represents α for $\nu = 2$.

from peaks of height 2 or 3 σ (BBKS; Ryden & Gunn 1987): so we repeated the

calculation of α for these values. For $\nu = 3$, the logarithmic slope of the density profile, calculated at $1h^{-1}Mpc$ is steeper for all values of n (solid line) than that obtained by HS, and it is well approximated by Shet & Jain (1996) (dotted line), $\alpha = \frac{3(4+n)}{(5+n)}$, obtained using stable clustering and neglecting halo-halo correlations. At the same time the dependence of α on n is weaker than that shown by HS. For

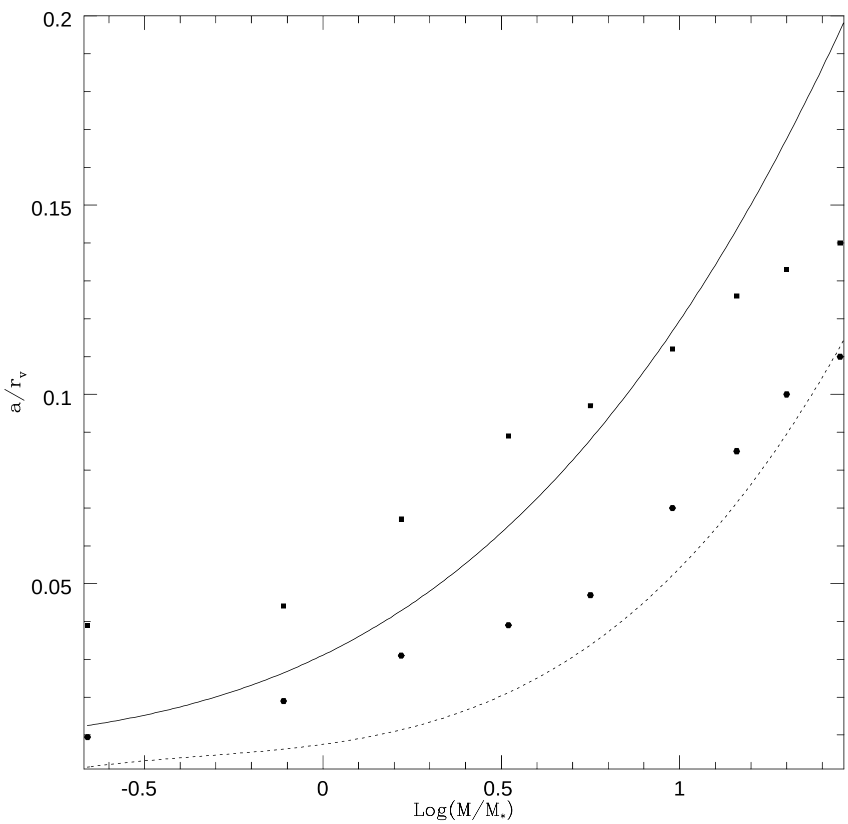

Figure 2: Trend of the scale radius a versus the mass of the halos in the case $n = -1$ (solid line) and $n = 0$ (dotted line). The filled squares and the filled exagons represents a/r_v, for $n = -1$ and $n = 0$ respectively, obtained by Navarro et al. (1997) in the case $f = 0.01$.

$\nu = 2$ the slope is even steeper than the previous case (long-dashed line) and it is well approximated by Crone's et al. (1994) result, which is also consistent with the

results by Navarro et al. (1997) (see their Fig. 13). As in the previous case the dependence of α on n is weaker with respect to that shown in HS.
In Fig. 2 we plot the variation of the scale parameters a versus M/M_*. Masses are normalized by the characteristic mass M_*, which is defined at a time t as the linear mass on the scale currently reaching the non-linear regime:

$$M_*(t) = \frac{4\pi}{3} R_*^3 \rho_b(t) \tag{15}$$

where the scale R_* is such that the linear density contrast on this scale is $\delta(R_*) = 1.69$ and $\rho_b(t)$ is the background density at the time t. Once known that the mass variance, σ_M, for a power spectrum $P(k) \propto k^n$ is given by $\sigma_M \propto R^{-(3+n)}$ and remembering our normalization $\sigma_M(8h^{-1}Mpc) = 0.63$, the value of M_* for $n = -1$ results to be $M_* = 6 \times 10^{13} M_\odot$. The value of the dimensionless scale radius a correlates strongly with halo mass and with spectral index n. The solid line represents a for $n = -1$. As shown in Fig. 2, more massive halos have a larger scale radius a, or equivalently less massive halos are more concentrated. The dotted line shows a for $n = 0$. Also for this value of n more massive halos are less centrally concentrated. Finally from Fig. 2 we also see that in models with more small-scale power (or equivalently larger values of n) the haloes tend to have denser cores. These results were expected because halos with mass $M << M_*$ form much earlier than haloes with $M >> M_*$ and then are more centrally concentrated. Moreover, for a fixed value of M/M_*, haloes form earlier in models with larger values of n and then have denser cores. This result is in qualitative agreement with those by Navarro et al. (1997), Cole & Lacey (1996), Tormen et al. (1997). The filled squares and the filled exagons, in Fig. 2, represents a/r_v for $n = -1$ and $n = 0$ respectively obtained by Navarro et al. (1997) in the case $f = 0.01$ (see their paper for a definition of this parameter) which give the best fit to the results of their simulations.
The virial radius r_v is obtained by using Navarro et al. (1997) equation and it determines the mass of the halo through:

$$M_v = 200\rho_c \frac{4\pi}{3} r_v^3 \tag{16}$$

In the case $n = -1$, our model gives less concentrated halos till $M \simeq 10M_*$ and after this value the tendence is reversed. In the case $n = 0$, our model gives halos sligthly more concentrated in the overall studied mass range.
The result obtained is remarkable. Our model is based on spherical simmetry, and as we previously stressed, halos accretion does not happen in spherical shells but by

aggregation of subclumps of matter which have already collapsed. In other words it seems that the halos structures does not depend crucially on hierarchical merging, in agreement with Huss et al. (1998). The SIM seems to have more predictive power than that till now conferred to it.

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